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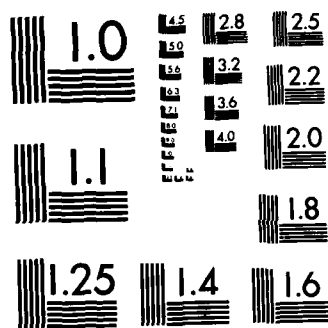
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TEAM THEORY AND DECENTRALIZED RESOURCE ALLOCATION:  
AN EXAMPLE

by  
KENNETH J. ARROW

TECHNICAL REPORT NO. 371  
February 1982

A REPORT OF THE  
CENTER FOR RESEARCH ON ORGANIZATIONAL EFFICIENCY  
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# TEAM THEORY AND DECENTRALIZED RESOURCE ALLOCATION: AN EXAMPLE\*

by

Kenneth J. Arrow

## 1. Introduction.

The traditional discussion of the price system and alternative forms of decentralized resource allocation in organizations and entire economies has an ambivalent attitude to the ease of transferring information from one locus in the economic system to another. On the one hand, the very need for decentralization is based on the assumption that the transmission of information is costly. If this were not so, there would be no reason not to transfer all information on the availability of resources and the technology of production to one place and compute at one stroke the optimum allocation of resources.<sup>1/</sup> On the other hand, the literature has tended to seek algorithms which, in some sense, minimize the amount of information transferred but which at the same time yield in the end the fully optimal allocation of resources.<sup>2/</sup> In short, there is no true trade-off between information costs and other resource costs. If there were, one would expect that an optimal allocation of resources, taking account of information costs, would differ from the optimal allocation in the absence of information costs. The standard tradition can be rationalized only by assuming that information costs are infinitesimal but not zero; hence, they should be minimized to the extent possible without affecting the overall allocation.<sup>3/</sup>

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The theory of teams was introduced by Jacob Marschak precisely to bring information costs into the allocation process explicitly; see J. Marschak [1955], J. Marschak and Radner [1972]. It does so in a way which is polar to the standard tradition. It assumes a fixed amount of communication in fixed channels. The "costs" of communication are modelled by scarcity.

Team theory differs in other ways from the standard approach. It makes more use of prior information about the economy. In the usual form of the price-adjustment or quantity-adjustment iterative processes for achieving an optimal allocation of resources, the rule design uses only the broadest qualitative information about the economy. There are no assumptions about the likely shapes of the production functions or the range of possible levels of resource supplies<sup>4/</sup> In team theory, some or all of the basic parameters of technology or resources are unknown; otherwise, there would be no informational problem at all. But there is prior information in the sense that probabilities are attached to different possible values of the parameters. The decisions made under decentralization can then take advantage of this knowledge and reduce the probability of a bad decision as far as possible.

There is still another difference between the standard approach and team theory, which follows from the fact that there are irreducible differences in information among the members of the team. In the standard approach, the decisions are ultimately all made by the central authority, the individual parts being only sources of information. In a sense, since relevant information is eventually equalized, it does not

really matter who makes the decisions. In team theory, as in real life, the allocations are ultimately the results of many individual decisions. The decentralization is real.

One implication among others is that the rules must be designed to insure feasibility without a full exchange of information on resource supplies.

The general form of the team problem, then, is this: The system has a number of agents, each charged with making certain decisions. The system's operations are governed by a number of parameters, initially unknown to the agents. There is a probability distribution over these parameters, reflecting prior knowledge. Each agent is given some information on some of the parameters. That is, the probability distribution of the parameters is conditioned on the agents' items of information. Each agent then makes a decision within its competence as a function of the information available to it. The team problem is to choose, in advance, the rules or decision functions for all agents. These decision functions are determined jointly to optimize the expected outcome, which is a function of the true values of the parameters and of the decisions made.

I will not try here to repeat in more detail the general formulation of the team problem, which can be found in J. Marschak and Radner [1972]. The structure will be sufficiently clear from the specific example to be analyzed. This example is very simple but requires sufficient analysis to indicate the nature of the team theory approach to resource allocation and the problems that need to be solved in applying it.

We assume a known production structure, indeed, the simple one of fixed coefficients. What are unknown a priori are the resource supplies. Each resource manager knows the supply of his or her own resource, and is required to divide it between the general production using all supplies and a specialized alternative use. The paper is devoted primarily to establishing the decision rules for the resource managers which yield the optimal resource allocation for the given information structure.

A modification of the model would permit the resource managers to transmit incomplete information about resource holdings. In that case, the center, having received the information, would then issue decision rules to the resource managers. The extension is in fact very straightforward. The calculation of the benefits from the additional information is straightforward in principle but does not lend itself to simple expression in formulas. These benefits should be compared with the costs of the additional information.

## 2. Formal Statement of the Model.

We are assumed to have  $n$  resources. These resources can be used jointly to produce a product. The production structure is defined by fixed coefficients, i.e., each unit of output requires a fixed amount of each resource. By choosing units properly, we can assume that the fixed coefficients are all 1, i.e., that it requires 1 unit of each resource to produce 1 unit of the output. Let,

$y$  = output,

$x_i$  = amount of  $i^{\text{th}}$  resource available,

$x'_i$  = amount of  $i^{\text{th}}$  resource used in production of output.

Then,

$$(2.1) \quad y = \min (x'_1, \dots, x'_n) = \min_i x'_i .$$

Clearly, if the only use of the resources is in the production of this product, there is no interesting resource allocation problem. The entire amount  $x_i$  are delivered to the production unit; no resource manager should be affected by the fact that he or she does not know the amounts of the other resources. Notice that resources other than the ones in least supply are useless in production; but if there are no alternative uses, that does not matter.

To make the allocation interesting, therefore, assume that each resource has another use, with a unit value which is constant relative to the joint product. Let,

$w_i$  = unit value of resource in alternate use,

with joint product as numeraire .

The aim of the team is to achieve as large a value of,

$$y + \sum_{i=1}^n w_i (x_i - x'_i) ,$$

as possible.

If the resource availabilities  $x_i$  were known, this problem would have an easy solution. It must, of course, satisfy the feasibility constraints,

$$(2.2) \quad 0 \leq x_i \leq x'_i \quad \text{all } i .$$

The problem would have a totally trivial solution if,

$$\sum_{i=1}^n w_i \geq 1 ,$$

for in that case, it would always be at least as good to engage only in specialized use of the resources. To see this, consider any feasible allocation. Now consider a new one which simply shuts down the joint production. The value of the specialized uses of the resources increases by,  $\sum w_i x'_i$ , while the amount of the joint produce decreases by,  $\min_i x'_i$ . Hence, the net gain is,

$$\sum_{i=1}^n w_i x'_i - \min_i x'_i \geq \sum_{i=1}^n w_i \min_i x'_i - \min_i x'_i = \left( \sum_{i=1}^n w_i - 1 \right) \min_i x'_i \geq 0 .$$

I.e., there is never any loss in shutting down joint production. In this case, then, the decision rules for the individual resource managers are simple: use all the resources for the specialized use. This rule can be carried out by the resource managers without any knowledge of each other's resources availabilities.

Hence, we assume,

$$(2.3) \quad \sum_{i=1}^n w_i < 1 .$$

If there is complete information, then clearly the optimal policy would be to set,  $x_i' = \min_j x_j$  for each  $i$ . That is, we would want to invest as much as possible of each resource in the general production process, subject to the non-wastefulness condition that an equal amount of each resource be used. This is because increasing the allocation of each resource to the general process by 1 unit, if feasible, increases output of the community by 1 and decreases the total value in the specialized processes by  $\sum w_i$ .

This allocation, however, requires that each resource manager know the amounts of all other resources (or at least the minimum of all other resource availabilities). Let us state explicitly our decision and information assumptions. Beforehand, when the rules are to be determined, each of the variables  $x_i$  is assumed to be a random variable, with a distribution known to all. For simplicity, I will assume that these variables are independent of each other. The resources become available, but for each resource only the manager knows the availability. Because of the independence assumption, no manager has any more information about the availability of other resources than he or she did before the realization of the random variables. The manager has then to determine the amount allocated to the general production process, in accordance with the rules laid down.

The allocation that finally results depends on the resources actually available; it must in fact satisfy the resource feasibility conditions (2.2). But it must also satisfy what may be termed the conditions of informational feasibility: a decision cannot depend on more information than is available to the decision-maker.

The last condition, stated formally, is that  $x'_i$ , the amount of resource  $i$  devoted to general production, can depend only on  $x_i$ , the amount of resource  $i$  available, for this is all the information available to the manager for resource  $i$ . The allocation  $x'_i$  cannot depend on the availabilities of any resources  $x_j$  for  $j \neq i$ . The team allocation problem then requires the choice of function,  $x'_i(x_i)$ , satisfying (2.2).

$$(2.4) \quad 0 \leq x'_i(x_i) \leq x_i, \text{ for all values of } x_i.$$

Finally, what is the maximand of the team problem? Since we regard the availabilities,  $x_i$ , as random variables, we must seek to maximize the expected payoff for each individual realization,

$$y + \sum_{i=1}^n w_i (x_i - x'_i) = \min_i x'_i(x_i) + \sum_{i=1}^n w_i [x_i - x'_i(x_i)] .$$

Therefore, we can finally state the team version of the allocation problem as follows:

Choose  $n$  functions,  $x'_i(x_i)$ , the  $i^{\text{th}}$  function depending on the quantity  $x_i$  alone, so as to maximize,

$$(2.5) \quad W = E\{\min_i x'_i(x_i) + \sum_{i=1}^n w_i [x_i - x'_i(x_i)]\} ,$$

subject to the constraints (2.4).

It may be useful to remark that the standard price-adjustment and quantity-adjustment procedures amount to full information revelation in the present context. Since resource supplies are inelastic, at the very first announcement of prices, the full set of supplies is announced in

the price-adjustment procedure. As for the quantity-adjustment procedure, this depends on the center's knowing how much of each resource is available and hence presupposes full information. Thus, in the present simple example, the standard methods of decentralized resource allocation require full information by the end of the first round of approximations.

### 3. The Form of the Optimal Team Policy.

Optimizing on functions is intrinsically more difficult than optimizing on variables; and the problem is complicated here by the fact that the functions are defined over specified variables, differing from one to another. In general, therefore, team theory problems are difficult to solve. But in this case, the functions can be shown to have some relatively simple forms, and the problem can be reduced to an optimization over ordinary real-valued variables.

If we have chosen our policy optimally, then it must also be that the decision rule for any one manager must be optimal, given the rules for all other managers. Consider manager 1. The policies,  $x_i'(x_i)$ , of all managers with  $i > 1$  are taken as given. Since  $x_1$  is independent of  $x_i$ , for  $i > 1$ , it is also true that  $x_i'(x_i)$  is independent of  $x_1$  for  $i > 1$ . Hence, manager 1 may regard the allocations,  $x_i'$ , of other managers as given random variables, independently of the observed variable,  $x_1$ .

Observe also that, to choose an optimal function or rule,  $x_1'(x_1)$  is really the same as choosing an optimal value for  $x_1'$  for each given value of  $x_1$ , since  $x_1$  is a known magnitude to manager 1.

We will rewrite (2.5) so as to separate the terms involving  $x'_1$  from the others, over which manager 1 has no influence. We use the fact that the expectation of a sum is the sum of the expectations. First, write

$$(3.1) \quad W = E[\min_i x'_i - \sum_{i=1}^n w_i x'_i] + E[\sum_{i=1}^n w_i x_i] \quad .$$

The last term is independent of any manager's actions; it is just a constant which does not affect the choice of the optimal team decision rules. Therefore, write the first terms separately:

$$(3.2) \quad W_1 = E[\min_i x'_i - \sum_{i=1}^n w_i x'_i] \quad .$$

It is  $W_1$  that is to be maximized by choice of the decision functions,  $x'_i(x_i)$ .

Now, to consider the optimization problem for manager 1 alone, write,

$$x'_{-1} = \min_{i>1} x'_i \quad .$$

Then,

$$\min_i x'_i = \min(x'_1, x'_{-1}) \quad .$$

Write (3.2) as follows:

$$(3.3) \quad W_1 = E[\min(x'_1, x'_{-1})] - w_1 x'_1 - E[\sum_{i=2}^n w_i x'_i] \quad .$$

From the viewpoint of manager 1, the values of  $x'_i$  for  $i > 1$  are given random variables. Hence, the third term is a constant; in the first term,  $x'_{-1}$  is taken as a given random variable. Finally,  $x_1$  is known to manager 1. It does not enter explicitly in (3.3) at all, but it does set an upper bound to the choice of  $x'_1$ , through the feasibility constraint (2.2) for  $i = 1$ :

$$(3.4) \quad 0 \leq x'_1 \leq x_1 .$$

The manager's aim is to choose  $x'_1$  to maximize (3.3) subject to (3.4). In technical terms, (3.3) is a concave function of the decision variable  $x'_1$ , that is, the marginal contribution of  $x'_1$  decreases. We use a well-known general result, recapitulated in the Appendix to this article: If  $x$  is any random variable and  $x^*$  a number to be chosen, then,

$$(3.5) \quad \frac{dE[\min(x^*, x)]}{dx^*} = \text{Prob}(x > x^*) .$$

Apply this to the differentiation of (3.3), with  $x^*$  replaced by  $x'_1$ ,  $x$  by  $x'_{-1}$ :

$$\frac{dW_1}{dx'_1} = \text{Prob}(x'_{-1} > x'_1) - w_1 .$$

As  $x'_1$  increases, the probability of exceeding it necessarily decreases. As  $x'_1$  becomes very small,  $\text{Prob}(x'_{-1} > x'_1)$  tends to 1, so that  $dW_1/dx'_1$  tends to  $1 - w_1$ ; in view of assumption (2.3), this must be positive. As  $x'_1$  becomes large,  $\text{Prob}(x'_{-1} > x'_1)$  approaches 0, so that  $dW_1/dx'_1$  approaches  $-w_1 < 0$ . Hence, there is an intermediate

point, say  $x_1^*$ , at which the function  $W_1$  achieves a maximum. It is increasing up to  $x_1^*$  and decreasing after that.

This gives the unconstrained maximum for  $W_1$ . However, we have to take account of the upper bound constraint, (3.4). Clearly, if  $x_1 > x_1^*$ , Then the manager should choose  $x_1^*$ , the unconditional maximum; while if  $x_1 < x_1^*$ , the manager wants to choose  $x_1$ , since  $W_1$  is increasing up to that point. In summary, the optimal decision rule for manager 1 has the form,

$$x_1'(x_1) = \min (x_1^*, x_1) \quad .$$

That is, there is a fixed number  $x_1^*$  independent of  $x_1$  which defines the optimal rule.

Since the same argument holds for any manager, not merely the first, we have established that the optimal decision rules have form,

$$(3.6) \quad x_i'(x_i) = \min (x_i^*, x_i) \quad .$$

These results show that the problem of choosing  $n$  functions,  $x_i'(x_i)$ , has been reduced to that of choosing  $n$  numbers,  $x_i^*$ .

We will substitute (3.6) into (3.5) and then optimize with respect to the parameters  $x_i^*$ . Because of (3.6), the feasibility constraints (2.4) are automatically satisfied. To simplify the resulting expression, introduce the new notation,

$$x^* = \min_i x_i^* \quad ,$$

$$x = \min_i x_i \quad .$$

Note that  $x$  is a random variable of known distribution, since it is the minimum of  $n$  independent random variables whose distributions are known. Then, from (3.6),

$$\min_i x'_i(x_i) = \min_i \min(x_i^*, x_i) = \min_i (\min x_i^*, \min x_i) = \min(x^*, x) .$$

$$W_1 = E[\min(x^*, x) - \sum_{i=1}^n w_i \min(x_i^*, x_i)] .$$

Suppose the  $x_i^*$ 's are not all equal, say,  $x_1^* > x^*$ . By definition of  $x^*$ , a small decrease in  $x_1^*$  will leave  $x^*$  unchanged, but it will decrease  $\min(x_1^*, x_1)$  for at least some values of  $x_1$ . The change therefore increases the expression in brackets for some values of the random variables and decreases it for none, so that  $W_1$  is increased. Hence at the optimum,  $x_1^* = x^*$ , and, in general,  $x_i^* = x^*$  for all  $i$ .

The optimization problem has now been reduced to the choice of a single number. Choose  $x^*$  to maximize,

$$(3.7) \quad W_1(x^*) = E[\min(x^*, x) - \sum_{i=1}^n w_i \min(x_i^*, x_i)]$$

$$= E[\min(x^*, x)] - \sum_{i=1}^n w_i E[\min(x_i^*, x_i)] .$$

Then the optimal decision rule for the  $i^{\text{th}}$  manager is,

$$(3.8) \quad x'_i(x_i) = \min(x^*, x_i) .$$

#### 4. The Determination of the Optimal Decision Rule.

It remains then only to determine  $x^*$  so as to maximize (3.7). The procedure is straightforward; the derivative of  $W_1$  with respect

to  $x^*$  is equated to zero. From (3.5), we see how to differentiate each of the terms in (3.7).

$$\frac{dW_1}{dx^*} = \text{Prob}(x > x^*) - \sum_{i=1}^n w_i \text{Prob}(x_i > x^*) .$$

For simplicity, introduce the following notation:

$$P(y) = \text{Prob}(x > y) ,$$

$$P_i(y) = \text{Prob}(x_i > y_i) .$$

These functions are like the well-known cumulative distribution functions, except that they are cumulated from above rather than below. Also, let

$$W'_1(x^*) = \frac{dW_1}{dx^*} .$$

Then,

$$W'_1(y) = P(y) - \sum_{i=1}^n w_i P_i(y) .$$

We now use the assumption that the random variables  $x_i$  are independent to express  $P(y)$  in terms of the probabilities  $P_i(y)$ . Indeed, the assertion,  $x > y$ , is by definition, equivalent to the assertion,  $\min x_i > y$ , and this in turn, by the definition of a minimum, holds if and only if  $x_i > y$  for all  $i$ . But the events,  $x_i > y$ , for any fixed  $y$ , are independent events, so the probability of their joint occurrence is the product of the probabilities of their individual occurrences. In symbols,

$$P(y) = \text{Prob}(x > y) = \text{Prob}\left(\min_i x_i > y\right) = \text{Prob}(x_i > y \text{ for all } i)$$

$$= \prod_{i=1}^n \text{Prob}(x_i > y) = \prod_{i=1}^n P_i(y) ,$$

where the symbol,  $\prod_{i=1}^n$ , means, "product as  $i$  runs from 1 to  $n$ ." The hypothesis of independence is used only in the fourth equality. We have then,

$$(4.1) \quad W'_1(y) = \prod_{i=1}^n P_i(y) - \sum_{i=1}^n w_i P_i(y) .$$

We know that  $x^*$  satisfies the condition,  $W'_1(x^*) = 0$ , but we need to show that such a root exists and in fact gives the maximum. In particular, it should be unique, at least in some relevant range; otherwise the equation may have several solutions, only one of which is the true optimum.

Let  $\bar{x}$  be the smallest value of  $y$  such that  $P_i(y) = 0$  for some  $i$ . It is obvious that one would never want to set  $x^*$  higher than that, for the contribution of the resource for which  $P_i(\bar{x}) = 0$  will never be more than  $\bar{x}$ , and any allocation of any other resource beyond  $\bar{x}$  will be certainly wasted. In the Appendix, the following is demonstrated: within the range from 0 to  $\bar{x}$ , there is in ordinary circumstances precisely one value of  $x^*$  for which,

$$W'_1(x^*) = 0 ,$$

where  $W'_1$  is defined in (4.1), and this value of  $x^*$ , used in (3.8), defines the optimal team rules for the managers.

5. Remarks on Operating Characteristics and Partial Information Transfer.

Given the optimum policy, it is always possible to determine its performance by substituting back into the expected payoff function,  $W$ , as given by (2.5). This can easily be done numerically in any given case, but no simply interpretable general formula seems available.

Within the framework of the problem as formulated, there is no especial need to determine the performance or operating characteristics of the team decision rule. However, it would be very useful if one were to contemplate alternative team descriptions, in particular the introduction of partial or total information transfer. In the latter case, one could calculate the operating characteristics for complete information, where the optimal policy is to set,

$$x'_1 = \min_j x_j ,$$

as noted before. Of course, this rule will give a higher expected payoff. However, there is presumably a cost to the provision of complete information, and this should be compared with the gain in payoff. It is for this reason that one would want to compute operating characteristics.

We consider briefly the case of partial information transfer. Suppose each resource manager provides the center with a random variable which is a signal for the availability of that resource. By a signal will be meant a random variable whose distribution is conditional on the actual value of the resource but independent of anything else, in

particular of the availabilities of other resources. By Bayes' Theorem, then, the center can compute a conditional distribution of the resource availability,  $x_i$ , for each  $i$ , which differs in general from the prior distribution. The Center can compute  $x^*$  as before but using the conditional distributions, and transmit that value as an instruction to each of the resource managers. The new ceiling on allocations to general production,  $x^*$ , is now a function of the signals transmitted. In the extreme case where the transmitted signal is  $x_i$  itself, we have complete information, and the value of  $x^*$  transmitted will be the optimal value under complete information.

An important but difficult optimization problem is the choice of signals, in other words, of the amount of information transfer to the center. Clearly some signals are more informative than others; that is, they convey knowledge of the resource availabilities more or less accurately.<sup>5/</sup> Hence, use of a more informative signal will yield a higher expected payoff. Presumably, however, more informative signals will tend to be more expensive. Hence, there is a need for comparing the expected gains with the expected additional costs. While the expected gains can be computed in any given case, as already seen, little apparently can be asserted of a general nature.

# Appendix

The proofs of two assertions were left to this Appendix.

The first is (3.5). Let  $x$  be any random variable and  $x^*$  any fixed number. We seek the derivative of  $E[\min(x^*, x)]$  with respect to  $x^*$ .

An expectation is essentially an integral. We can differentiate it by differentiating under the integral sign. Now  $\min(x^*, x) = x^*$  if  $x > x^*$  and  $= x$  if  $x < x^*$ . Hence

$$\begin{aligned} \frac{d \min(x^*, x)}{dx^*} &= 1 \quad \text{if } x > x^* , \\ &= 0 \quad \text{if } x < x^* . \end{aligned}$$

The derivative is undefined when  $x = x^*$ . However, if, as we have been implicitly assuming,  $x$  is a continuous random variable, the probability that it takes on any particular value,  $x^*$ , is zero, so the lack of definition of the derivative at one point does not affect the expectation.

We also note that if a function of a random variable is 1 on some range of  $x$ -values say the set  $E$ , and 0 elsewhere, then the expectation of that function is the probability of  $E$ . Hence,

$$\frac{dE[\min(x^*, x)]}{dx^*} = E\left[\frac{d \min(x^*, x)}{dx^*}\right] = \text{Prob}(x > x^*) ,$$

as asserted.

The second assertion left for proof in this Appendix is that the function,  $W_1'(y)$ , as defined in (4.1), has precisely one zero as  $y$

varies from 0 to  $\bar{x}$ , where  $\bar{x}$  is the smallest value of  $y$  for which  $P_i(y) = 0$  for some  $i$ . (It is not excluded that  $P_i(y) > 0$  for all  $i$  and all  $y$ , in which case we may take  $\bar{x}$  to be infinite.)

For  $y < \bar{x}$ ,  $P_i(y) > 0$  for all  $i$ , and therefore,

$$(A.1) \quad \prod_{i=1}^n P_i(y) > 0.$$

Let,

$$(A.2) \quad V(y) = \frac{W_1(y)}{\prod_{j=1}^n P_j(y)} = 1 - \sum_{i=1}^n w_i \frac{P_i(y)}{\prod_{j=1}^n P_j(y)} = 1 - \sum_{i=1}^n \frac{w_i}{\prod_{j \neq i} P_j(y)}.$$

From the definition of  $P_i(y)$ , as the probability that a random variable is greater than  $y$ , it must be true that  $P_i$  is monotone decreasing as  $y$  increases. It is also true that  $P_i$  is non-negative and in fact positive so long as  $y < \bar{x}$ . Clearly, then, the product of any number of  $P_i$ 's is positive and monotone decreasing. In particular, for each  $i$ , the function,

$$\prod_{j \neq i} P_j(y),$$

is positive and monotone decreasing. Hence,  $V(y)$  is monotone decreasing. We also observe that  $P_j(0) = 1$ , from the definition (resource availability is certainly positive), so that

$$V(0) = 1 - \sum w_i > 0.$$

As  $y$  approaches  $\bar{x}$ ,  $P_j(y)$  approaches 0 for one or more values of  $j$ , so that, from (A.2),  $V(y)$  approaches negative infinity. Therefore,  $V(y)$  must equal zero at least once in the interval from 0 to  $\bar{x}$ .

Since  $V(y)$  is monotone decreasing, it would ordinarily be true that it has exactly one zero. Strictly speaking, however, it is conceivable that a monotone decreasing function is zero on a whole interval.

Thus  $V(y)$  is positive for  $y$  small, 0 at a point or on an interval, and then negative. By construction,  $W'_1(y)$  has the same sign (positive, zero, or negative) as  $V(y)$ . Hence,  $W_1$  increases for small  $y$ , reaches a maximum, which may be at a point or over an interval, and then decreases.

Therefore any solution of the equations,  $V(y) = 0$ , or  $W'_1(y) = 0$ , may be taken as the value of the operating parameter,  $x^*$ , which determines the optimal team decision policy.

Footnotes

- 1/ For the purposes of this paper, I assimilate computation and its costs to information transfer. The economics of computation has so far not been well integrated with that of information. For an early and interesting essay, see T. Marschak [1959].
- 2/ Actually, there is rarely any explicit measurement of the information costs of alternative algorithms for achieving optimal resource allocation. Most of the work is satisfied with exhibiting a process which manifestly requires a good deal less than complete information transfer. For an explicit comparison of alternative algorithms according to the amount of information transfer needed, see Oniki [1974].
- 3/ What I have called the standard tradition of decentralized achievement of optimal resource allocation goes back to Adam Smith's "invisible hand," and continues through Walras [1954, pp. 84-86, 90-91, 105-106, 169-172, 243-254, and 184-195], Pareto [1927, pp. 233-234], Barone, pp. 245-290 in von Hayek [1935], J. Marschak [1924], Lange, pp. 57-142 in Lippincott [1938], and the more modern literature surveyed in Hurwicz, pp. 3-37 in Arrow and Hurwicz [1977]. Most of this literature has emphasized the parametric role of prices and the successive adjustment of prices in accordance with excess demands and supplies. A variant strain has emphasized quantity allocations and their revision so as to bring the shadow prices of each resource in different uses into equality; see especially Kornai and Liptak [1965], Marglin [1969], and Arrow [1976]. A very good survey of the entire literature is found in Heal [1973].

It should be added that Hurwicz, pp. 393-412 in Arrow and Hurwicz [1977], and other papers has given a more abstract version of the process of successive adjustments which includes the price-adjustment and quantity-adjustment types of algorithms as special cases. However, it has in common with the rest of the literature that information costs do not appear explicitly and that the process seeks a fully optimal allocation disregarding such costs.
- 4/ Indirectly, however, knowledge of the economy may be used to determine the starting point of the iterative process. In practice, of course, the length of the iterative process may be very much reduced by a suitable initial condition.
- 5/ The relation between signals, "more informative than," is definable in different ways. In general, it cannot be an ordering, that is, given any two signals, it is not necessarily true that one is more informative than the other. The most general useful definition is that of Blackwell [1953]. With this definition, it is always true that a more informative signal will yield a higher expected payoff.

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